NON-STEADY-STATE FLOW AT A POROUS PLATE WITH BLOWING (SUCTION) OF THE MEDIUM

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From the point of view of a generalization of [1], the present article touches on the not fully established boundary layer of the flow, formed in a viscous incompressible homogeneous liquid, surrounding an infinite porous plate, with uniform blowing or suction of the medium. The liquid and the plate are rotating as a solid body with a constant angular velocity ω_0 = const. Not fully established flow is induced by noncircular vibrations of the plate. The structure of the not fully established field of the velocities and of the boundary layer formed adjacent to the plate is determined. It has been found that, in this case (with blowing or suction of the medium), an exact solution can be found to the three-dimensional, non-steady-state Navier–Stokes equations. The velocity field found can be used for an analysis of an non-steady-state layer at the porous surface of a moving body. The non-steady-state problem without blowing has been discussed in [2].

1. Let us consider an infinite plate, moving with the velocity $\mathbf{u}(t)$ in its own plane. There is blowing or suction of the medium through the surface of the plate with the velocity $u_1(t)$. The medium is a viscous incompressible liquid, occupying a half-space bounded by the plate. The motion of the liquid is described by the Navier–Stokes equations and the following conditions, which, in the usual notation, have the form

$$\partial\mathbf{v}/\partial t + (\mathbf{v}\nabla)\mathbf{v} + 2\boldsymbol{\omega}_0 \times \mathbf{v} = -\nabla P + \nu\Delta\mathbf{v}, \operatorname{div}\mathbf{v} = 0 \text{ in } Q,$$
$$\mathbf{v} = \{\mathbf{u}(t),\ u_1(t)\mathbf{e}_y\} \text{ at } S,\ t > 0,\ |\mathbf{v}| \to 0 \text{ with } |\mathbf{r}| \to \infty,\ t > 0, \quad (1.1)$$

where $\mathbf{e}_y$ is a unit vector of the Cartesian system of coordinates Oxyz, perpendicular to the plane of the plate. The plane Oxz coincides with the plane of the plate.

The motion of the liquid starts from a state of rest, so that $\mathbf{v}(0, \mathbf{r}) = 0$ with $\mathbf{r} \in \bar{Q}$.

We seek the solution of system (1.1) in the form

$$\mathbf{v} = \{v_x(y,\ t),\ u_1(t),\ v_z(y,\ t)\},$$
$$P = 2\omega_{0z}u_1(t)x - 2\omega_{0x}u_1(t)z - \partial u_1 y/\partial t + p(y,\ t).$$

To determine the field of the velocities we obtain the following system of equations:

$$\partial v_x/\partial t + 2\omega_{0y}v_z = Lv_x,$$
$$\partial v_z/\partial t - 2\omega_{0y}v_x = Lv_z,$$
$$\partial p/\partial y = 2(\omega_{0z}v_x - \omega_{0x}v_z), \quad (1.2)$$

where $L = \nu\partial^2/\partial y^2 - u_1(t)\partial/\partial y$.

We seek the solution of system (1.2) in the form

$$\mathbf{v} = \mathbf{w}\cdot \sin 2\Omega t - \mathbf{w} \times \mathbf{e}_y\cdot\cos 2\Omega t, \quad (1.3)$$

where $\mathbf{w}$ is an unknown function; $\Omega = \omega_{0y}$.

The unknown function $\mathbf{w}(y, t)$ satisfies a differential equation of parabolic type and the boundary conditions

$$\partial\mathbf{w}/\partial t = L\mathbf{w};$$
$$\mathbf{w} = \mathbf{u}(t)\cdot \sin 2\Omega t + \mathbf{u}(t) \times \mathbf{e}_y\cdot\cos 2\,\Omega t,$$
$$\text{with } \mathbf{r} \in S,\ t > 0; \quad (1.4)$$

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$$|\mathbf{w}| \to 0 \text{ with } t = 0,\ y > 0;$$
$$|\mathbf{w}| \to 0 \text{ with } y \to \infty.$$

Let us first examine the case where $u_1(t) = \text{const} = a$, which corresponds to uniform suction or blowing. Here, $a > 0$ corresponds to blowing of the medium through the surface of the porous plate, and $a < 0$ to suction of the surrounding medium.

The solution of the problem (1.4) using a Duhamel integral can be written in the form

$$\mathbf{w}(y, t) = \frac{d}{dt}\int_0^t [\mathbf{u}(\tau)\cdot\sin 2\Omega\tau + \mathbf{u}(\tau)\times\mathbf{e}_y\cdot\cos 2\Omega\tau]\,|\mathbf{w}_1(y, t-\tau)|\,d\tau, \tag{1.5}$$

where $\mathbf{w}_1$ is a solution of the following boundary-value problem:

$$\begin{gathered} \partial\mathbf{w}_1/\partial t + a\partial\mathbf{w}_1/\partial y = \nu\partial^2\mathbf{w}_1/\partial y^2, \\ \mathbf{w}_1(0, t) = \begin{cases} 1, & \text{for } t > 0, \\ 0, & \text{for } t < 0, \end{cases} \\ |\mathbf{w}_1| \to 0 \text{ with } y \to \infty. \end{gathered} \tag{1.6}$$

We solve the system of equations (1.6) using the method of a Laplace transform. We introduce the Laplace transform of the functions by the relationship

$$\tilde{\mathbf{u}}(p) = \int_0^\infty e^{-pt}\cdot\mathbf{u}(t)\,dt.$$

In the space of the transforms, the differential equation (1.6) has the form

$$\begin{gathered} p\tilde{\mathbf{w}}_1 + a\partial\tilde{\mathbf{w}}_1/\partial y = \nu\partial^2\tilde{\mathbf{w}}_1/\partial y^2, \\ \tilde{\mathbf{w}}_1|_{y=0} = (1/p)\cdot\mathbf{1}, \\ |\tilde{\mathbf{w}}_1| \to 0 \text{ with } y \to \infty. \end{gathered} \tag{1.7}$$

The solution of system (1.7) has the form

$$\begin{gathered} \tilde{\mathbf{w}}_1(p, y) = (1/p)\cdot\mathbf{1}\cdot e^{-\lambda y},\ \operatorname{Re}\lambda > 0, \\ \lambda = a/2\nu + \sqrt{a^2/4\nu^2 + p/\nu}. \end{gathered} \tag{1.8}$$

Going over to inverse transforms, the solution of (1.8) can be written in the form [3]

$$\mathbf{w}_1(y, t) = \frac{1}{2}\,\mathbf{1}\cdot e^{-ay/2\nu}\left[e^{-ay/2\nu}\operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} - \frac{a}{2}\sqrt{\frac{t}{\nu}}\right) + e^{ay/2\nu}\operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} + \frac{a}{2}\sqrt{\frac{t}{\nu}}\right)\right]. \tag{1.9}$$

Thus, the solution of system (1.4) is given by formulas (1.5) and (1.9).

Substituting (1.5) into (1.3), taking account of (1.9), we obtain the sought velocity field of the starting problem

$$\begin{aligned} \mathbf{v} = {} & \sin 2\Omega t\cdot\frac{d}{dt}\int_0^t [\mathbf{u}(\tau)\cdot\sin 2\Omega\tau + \mathbf{u}(\tau)\times\mathbf{e}_y\cdot\cos 2\Omega\tau]\,\frac{1}{2}\left[e^{-ay/\nu}\operatorname{erfc}\left(\frac{y}{2\sqrt{\nu(t-\tau)}} - \frac{a}{2}\sqrt{\frac{t-\tau}{\nu}}\right) + \right. \\ & \left. + \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu(t-\tau)}} + \frac{a}{2}\sqrt{\frac{t-\tau}{\nu}}\right)\right] d\tau + \cos 2\Omega t\cdot\mathbf{e}_y\times\frac{d}{dt}\int_0^t [\mathbf{u}(\tau)\cdot\sin 2\Omega\tau + \\ & + \mathbf{u}(\tau)\times\mathbf{e}_y\cdot\cos 2\Omega\tau]\,\frac{1}{2}\left(e^{-ay/\nu}\operatorname{erfc}\frac{y}{2\sqrt{\nu(t-\tau)}} - \frac{a}{2}\sqrt{\frac{t-\tau}{\nu}}\right) + \\ & + \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu(t-\tau)}} + \frac{a}{2}\sqrt{\frac{t-\tau}{\nu}}\right)\Bigg]\,d\tau. \end{aligned} \tag{1.10}$$

2. Let us consider the practically important case of motion of a plate at a constant

acceleration. It is convenient to carry out further transformation in complex form. We introduce the complex vectors

$$\hat{v} = v_z + iv_x, \quad \hat{u} = u_z + iu_x. \tag{2.1}$$

Using (2.1) and setting $a = 0$ (no blowing), the field of the velocities can be rewritten in the form

$$\hat{v} = \frac{y}{2\sqrt{\pi\nu}} \int_0^t \frac{\hat{u}(\tau)}{(t-\tau)^{3/2}} e^{-i2\Omega(t-\tau)-y^2/4\nu(t-\tau)} d\tau.$$

It is of interest to determine the viscous stresses acting on the plate from the side of the plate:

$$\hat{f} = -\rho\sqrt{\frac{\nu}{\pi}} \int_0^t \frac{\frac{\partial}{\partial\tau}\left[\hat{u}(\tau)\, e^{-i2\Omega(t-\tau)}\right]}{(t-\tau)^{1/2}} d\tau.$$

Setting $\hat{u} = \hat{b}t$, where $\hat{b}$ is a constant vector, we have

$$\hat{v} = \frac{\hat{b}y}{2\sqrt{\pi\nu}} \int_0^t \frac{\tau e^{-i2\Omega(t-\tau)-y^2/4\nu(t-\tau)}}{(t-\tau)^{3/2}} d\tau.$$

Denoting $\sigma = i2\Omega$, and carrying out the replacement of variables $t - \tau = \theta$; $\tau = t - \theta$, we obtain

$$\hat{v} = \frac{\hat{b}y}{2\sqrt{\pi\nu}} \int_0^t \frac{t-\theta}{\theta^{3/2}} \exp\left[-\sigma\theta - \frac{y^2}{4\nu\theta}\right] d\theta. \tag{2.2}$$

Dividing the integral in (2.2) into two parts, we obtain

$$J_1 - J_2 = \frac{\hat{b}yt}{2\sqrt{\pi\nu}} \int_0^t \frac{\exp\left[-\sigma\theta - \frac{y^2}{4\nu\theta}\right]}{\theta^{3/2}} d\theta - \frac{\hat{b}y}{2\sqrt{\pi\nu}} \int_0^t \frac{\exp\left[-\sigma\theta - y^2/4\nu\theta\right]}{\theta^{1/2}} d\theta;$$

J_1 was calculated earlier [2] in the form

$$J_1 = \frac{\hat{b}t}{2}\left[e^{-y\sqrt{\frac{\sigma}{\nu}}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} - \sqrt{\sigma t}\right) + e^{y\sqrt{\frac{\sigma}{\nu}}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} + \sqrt{\sigma t}\right)\right].$$

Taking account of the calculation of the similar integral in [2], and omitting the computational techniques, we give the expression for J_2

$$J_2 = \frac{\hat{b}y}{4\sqrt{\sigma\nu}}\left[e^{-y\sqrt{\frac{\sigma}{\nu}}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} - \sqrt{\sigma t}\right) - e^{y\sqrt{\frac{\sigma}{\nu}}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} + \sqrt{\sigma t}\right)\right].$$

For $\hat{v}$ we finally have

$$\hat{v} = \frac{1}{2}\hat{b}\left(t + \frac{y}{2\sqrt{\sigma\nu}}\right) e^{y\sqrt{\frac{\sigma}{\nu}}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} + \sqrt{\sigma t}\right) + \frac{1}{2}\hat{b}\left(t - \frac{y}{2\sqrt{\sigma\nu}}\right) e^{-y\sqrt{\frac{\sigma}{\nu}}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} - \sqrt{\sigma t}\right).$$

We introduce a characteristic parameter, the so-called complex time:

$$t^* = y/2\sqrt{\sigma\nu}.$$

Then

$$\hat{v} = \frac{1}{2}\hat{b}(t + t^*) e^{y\sqrt{\frac{\sigma}{\nu}}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} + \sqrt{\sigma t}\right) + \frac{1}{2}\hat{b}(t - t^*) e^{-y\sqrt{\frac{\sigma}{\nu}}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} - \sqrt{\sigma t}\right).$$

The velocity field obtained can be used for analysis of an non-steady-state boundary layer at the porous surface of a moving body.

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COMPLEX HEAT EXCHANGE OF A DISPERSED TURBULENT FLOW IN A PIPE

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1. We will write the energy equation for a turbulent flow of a gas and suspended matter in a pipe in the form [1, 2]

$$(1-\beta)c\rho u\frac{\partial T}{\partial x}+\beta c_1\rho_1 u_1\frac{\partial T_1}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left\{r(\lambda+\lambda_t)\frac{\partial T}{\partial r}-r\beta c_1\rho_1\langle v_1'T_1'\rangle\right\}+\operatorname{div}\mathbf{q}_r, \quad (1.1)$$

where β is the volume concentration of the solid phase, λ and λ_t are the molecular and turbulent thermal conductivity, respectively, $\langle v_1'T_1'\rangle$ is the turbulent energy transport by particles, and $\mathbf{q}_r$ is the resultant radiation flux.

It is necessary for a description of the heat exchange of a two-phase flow to supplement Eq. (1.1) with an energy equation for the particles. We will discuss the turbulent flow of a gas and suspended matter with a low concentration of heavy particles, i.e., $\beta \ll 1$. The particles are distributed uniformly over the cross section of the pipe. Then Eq. (1.1), upon neglect of terms containing β, is written in dimensionless form as

$$\frac{u}{\bar{u}}\frac{\partial\Theta}{\partial\bar{x}}=\frac{4}{\mathrm{Re}}\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left\{\bar{r}\left(\frac{1}{\mathrm{Pr}}+\frac{1}{\mathrm{Pr}_t}\frac{\nu_t}{\nu}\right)\frac{\partial\Theta}{\partial\bar{r}}\right\}+\frac{2}{\mathrm{Bo}}\operatorname{div}\bar{\mathbf{q}}_r, \quad (1.2)$$

where $\Theta = T/T_0$, $\bar{x} = x/D$, $\bar{r} = r/R$, $\mathrm{Bo} = c\rho\bar{u}/\sigma_0T_0^3$, $\mathrm{Re} = \bar{u}D/\nu$; $\bar{u}$ is the average velocity, Pr_t is the turbulent Prandtl number, T_0 is the ambient temperature at the pipe entrance ($T_0 > T_1$), and ν_t is the turbulent viscosity. The boundary conditions for (1.2) are of the form

$$\Theta(\bar{r};\,0)=1,\ \Theta(\bar{x}>0;\,1)=\Theta_1.$$

2. The velocity distribution in viscous and transition layers is given in [3] in the form

$$\frac{u}{v^*}=\frac{2.3}{\varkappa_0}\lg\frac{v^*y}{\nu}+5.8,\quad 30\leqslant\frac{v^*y}{\nu}\leqslant700;$$

$u/v^* = v^*y/\nu$, and $v^*y/\nu < 30$ (y is the distance from the wall).

The value of the tangential stress τ in a dispersed flow is related to the analogous quantity in a pure gas flow τ_0 by the relationship

$$\tau/\tau_0 = 1+\eta\mu, \quad (2.1)$$

where μ is the discharge concentration and η is a coefficient reflecting the strength of the effect of μ on the degree of deformation of the velocity distribution in a two-phase flow.

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